# Inductance Extraction for Planar Spiral Inductor by Domain Decomposition Method for Conformal Modules

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***Abstract*—In this paper the domain decomposition method (DDM) for conformal modules is used to get simple analytic expressions for inductance of planar spiral inductors. DDM is found to be valid for inductors of arbitrary spiral shape, as long as the conductor width is uniform along the spire. Expressions of a rectangular spiral inductor and a circular spiral inductor are obtained by DDM, and the computed results are compared with previously published expressions.**

## I. INTRODUCTION

Monolithic inductors and transformers, especially in the form of spirals, have gained much application in the design of integrated RF transmitters and receivers. For this reason, the analysis and optimization of such structures have been of great importance. Considerable research work has been done for the design and modeling of symmetrical structure inductor [1], [2], but for arbitrary shape inductors, the existed methods fail. In this paper the domain decomposition method (DDM) for conformal modules is used to get simple analytic expressions for inductance of planar spiral inductors with arbitrary spiral shape and uniform conductor width along the spire. DDM for conformal modules can avoid complex conformal mapping. It transforms directly a planar spiral to an elongate rectangular strip. As a result, the complex problem of computing the inductance of planar spiral inductors with arbitrary spiral shape and uniform conductor width along the spire is transformed to computing the inductance of the simple rectangular strip. By using DDM for conformal modules, the expressions of a rectangular spiral inductor and a circular spiral inductor are obtained. The computed results from the expressions are compared with previously published expressions and good agreement is achieved.

## II. INDUCTANCE EXTRACTION WITH DDM FOR CONFORMAL MODULES

At first the concept of conformal module of a quadrilateral is introduced. Let $Q := \{\Omega; z_1, z_2, z_3, z_4\}$ denote a quadrilateral consisting of a Jordan domain $\Omega$ and four specified points $z_1, z_2, z_3, z_4$ on boundary $\partial\Omega$. The conformal module $m(Q)$ of $Q$ is defined as follows. Suppose $R_h$ denotes a rectangle of the form $R_h := \{(\xi, \eta) : 0 < \xi < 1, 0 < \eta < h\}$, and then $m(Q)$ is the unique value of $h$ for which $Q$ is conformally equivalent to the rectangular quadrilateral $\{R_h; 0, 1, 1+ih, ih\}$. By this we mean that for $h = m(Q)$ and for this value only there exits a unique conformal map $F : \Omega \to R_h$, which takes the four points $z_1, z_2, z_3, z_4$ respectively onto the four vertices $0, 1, 1+ih, ih$ of $R_h$ [3].

Now suppose figure 1 represents a planar spiral inductor with arbitrary spiral shape and uniform conductor width $w$.

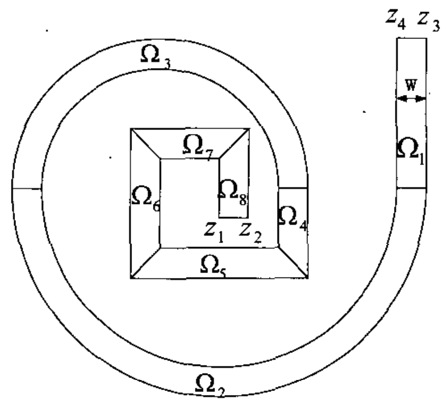


Fig.1. DDM for arbitrary shape inductor

The quadrilateral $Q$ is divided into $N$ sub-domains $Q_i$, $i = 1,\ldots,8$. The total conformal module of $Q$ is $m(Q) = \sum m(Q_i)$, where $m(Q_i)$ is the conformal module of the $i$th sub-domain $Q_i$. Let $T_i$ and $A$ denote the trapezium and half- circular annulus respectively

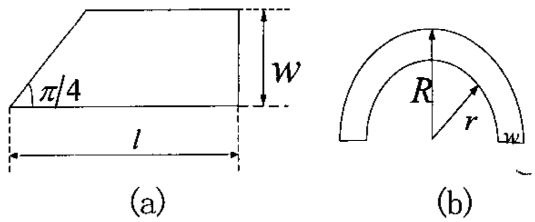


Fig. 2. (a) Trapezium sub-domain, (b) Half circular annulus sub-domain

(See Fig. 2.), then the conformal module $m(T_l)$ and $m(A)$ are expressed as below [3], [4]:

$$m(T_l) = \frac{2l - w}{2w} - \frac{1}{\pi}\log 2 \quad \text{For } l >> w \qquad (1)$$

$$m(A) = \frac{\pi}{\log R - \log r} \qquad (2)$$

The concept of conformal module is merely a mathematical concept. Its geometrical signification is the aspect ratio of the objective rectangle. However, in the process of calculating inductance, the width of the rectangle may be a certain data rather than 1. Fortunately, when $l >> w$ or $R > r >> w$ (as in Fig. 2), the equivalent width $w'$ in the transformational area can be approximately obtained by the parameters extraction method for conformal mapping, and the result is $w' \approx \pi w/3 + \log 8$. So the mapped rectangle is $[m(Q) \times w'] \times w'$. Note that, in the entire process, the thickness of conductor $t$ keeps constant. According to reference [5, p.35], the total inductance of the spiral inductor $Q$ is given by

$$L = \frac{\mu_0}{2\pi} w' m(Q)[\log(\frac{2w' m(Q)}{w' + t}) + 0.5 - \alpha] \qquad (3)$$

Where $\alpha$ is a modified factor, and a function of $r = [t/w' \text{ or } w'/t] \le 1$. We can obtain its expression by polynomial fitting from the table 3 in reference [5, p.22]

$$\begin{aligned} \alpha(r) \approx\ & 0.0718r^5 - 0.2157r^4 + 0.2457r^3 \\ & - 0.1300r^2 + 0.0299r + 1.4029 \times 10^{-4} \end{aligned} \qquad (4)$$

## III. APPLICATION EXAMPLES

In this section the accuracy of expression (3) is evaluated by comparison with previously published expressions through two examples.

*Example 1.* A square spiral inductor with $n = 3.5$ turns, total length $l = 2700\mu m$, spacing between neighbor conductor turns $s = 2\mu m$, thickness $t = 1\mu m$ and conductor width $w$ (see Fig.3). The inductor is divided into 15 sub-domains. The lengths of outer edge of the sub-domain $\Omega_{15}$ is

$$x = \frac{l - 39(s + w)}{28} \qquad (5)$$

From the formula (1), the $m(Q_i), i = 1, \cdots, 15$ are given as below:

$$m(Q_1) = \frac{2x + 7s + 6w}{2w} - \frac{1}{\pi}\log 2 \qquad (6)$$

$$m(Q_2) = 2(\frac{2x + 6s + 5w}{2w} - \frac{1}{\pi}\log 2) \qquad (7)$$

$$m(Q_{2i-1}) = m(Q_{2i}) = 2(\frac{2x + (7-i)s + (6-i)w}{2w} - \frac{1}{\pi}\log 2) \quad i = 2,..,7 \qquad (8)$$

$$m(Q_{15}) = \frac{2x - w}{2w} - \frac{1}{\pi}\log 2 \qquad (9)$$

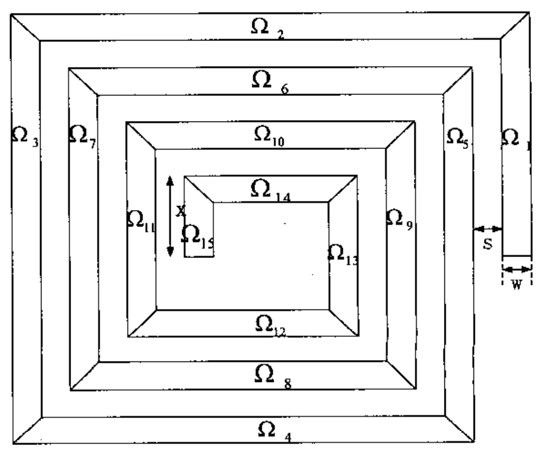


Fig.3. The square spiral inductor

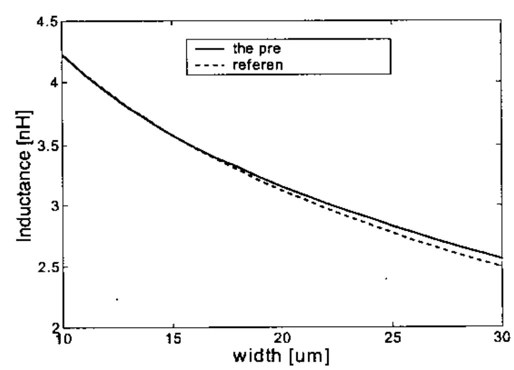


Fig.4. Comparison with reference [1] for a different conductor width $w$

The computed total inductance of the square spiral inductor as a function of the conductor width $w$ is depicted in Fig.4 and compared with the result in reference [1]. The reason of difference between the two methods is that the method in reference [1] neglects the modified factor $\alpha$.

*Example 2.* Consider the circular spiral inductor in Fig. 5 with $n = 3$ turns, total length $l = 2000\mu m$, and spacing between neighbor conductor turns $s = 4\mu m$. The entire spiral is divided into six sub-domains, the inner radius $r_i$ and outer radius $R_i$ of the $i$th sub-domain are given below:

$$\Omega_i : \begin{array}{l} r_i = 0.5(d_{in} + (i-1)(w+s)) \\ R_i = 0.5(d_{in} + 2w + (i-1)(w+s)) \end{array} \qquad (10)$$
$$i = 1, \ldots, 6$$

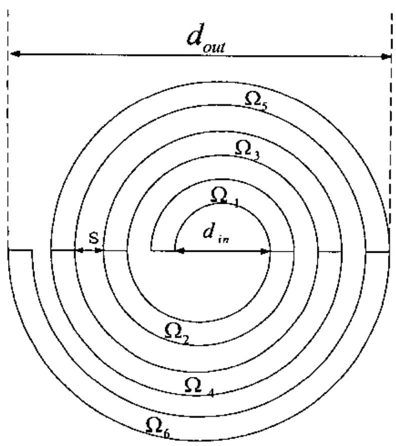


Fig. 5. The plane circular spiral inductor

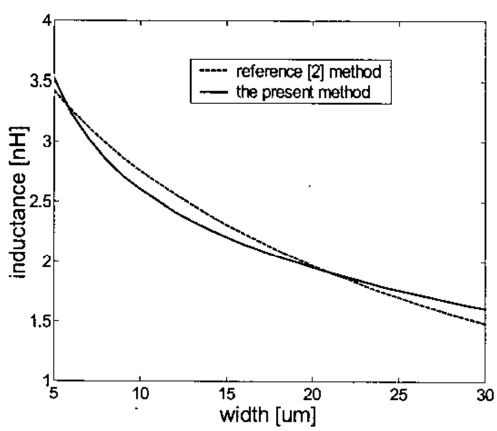


Fig.6. Comparison with reference [2] for a different conductor width

Where $d_{in}$ is the innerest diameter of the inductor,

$$d_{in} = \frac{(2l - 27\pi w - 15\pi s)}{6\pi} \qquad (11)$$

In order to compare with the results in reference [2], we assume the conductor thickness is $t = 0$. The computed inductance as a function of the metal width is depicted in Fig.6 and compared with that in reference [2]. The relative error is less than 8 percent.

## IV. CONCLUSION

Based on the domain decomposition method for computing conformal modules, an accurate, simple and closed form inductance formula has been presented in this paper. This method is valid for arbitrary spiral inductor as long as one can solve for the conformal module of every sub-domain. In addition, the obtained expression has the advantage that it contains both the self- and mutual- inductance, hence avoids tedious calculation for mutual inductance between every turn.


## ACKNOWLEDGEMENT.

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